

FTUV/96 – 34
IFIC/96 – 42

SUPERSYMMETRIC QUANTIZATION OF YANG-MILLS THEORY AND POSSIBLE APPLICATIONS ¹

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Abstract

We develop a new operator quantization scheme for gauge theories where no gauge fixing for gauge fields is needed. The scheme allows one to avoid the Gribov problem and construct a manifestly Lorentz invariant path integral that can be used in the non-perturbative domain. We discuss briefly an application of the method to Abelian projections of QCD.

1 Dynamics on the gauge orbit space

Quantum dynamics of Yang-Mills fields is described by the Hamiltonian

$$H_{YM} = \frac{1}{2} \int d^3x \left(\mathbf{E}_a^2 + \mathbf{B}_a^2 \right) , \quad (1)$$

where \mathbf{E}_a and \mathbf{B}_a are color electric and magnetic fields. Physical wave functionals must be gauge invariant $\Psi_D[\mathbf{A}^U] = \Psi_D[\mathbf{A}]$, $\mathbf{A}^U = U \mathbf{A} U^+ - iU \partial U^+$, $U^+ U = U U^+ = 1$, which is ensured by the Dirac conditions (the Gauss law)

$$\sigma_{YM}^a \Psi_D[\mathbf{A}] = \mathbf{D}^{ab} \mathbf{E}_b \Psi[\mathbf{A}] = 0 , \quad (2)$$

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with \mathbf{D}^{ab} being the covariant derivative in the adjoint representation. The operator σ_{YM}^a of the Dirac constraint is a generator of gauge transformations.

The Dirac states are functionals on the gauge orbit states $[\mathbf{A}]_{ph} = [\mathbf{A}]/G$ where $[\mathbf{A}]$ is the total configuration space and G is the gauge group, $G : \mathbf{A} \rightarrow \mathbf{A}^U$. In practical calculations, one needs to introduce local coordinates on the orbit space, for instance, to go over to the path integral formalism. In the conventional approach, one fixes a gauge, say, $F(\mathbf{A}) = 0$, to parametrize the orbit space by Cartesian "coordinates" satisfying the gauge condition. In this approach it is assumed that the subspace $[\mathbf{A}]_{F=0} \subset [\mathbf{A}]$ is isomorphic to the orbit space. This assumption was shown [1] to be not justified because the orbit space has a non-trivial topology, and therefore there is no global coordinate system on it. In other words, one cannot cover $[\mathbf{A}]_{ph}$ just by one Cartesian coordinate patch $[\mathbf{A}]_{F=0}$ without singularities.

A natural resolution of the problem would be to use many patches that cover $[\mathbf{A}]_{ph}$ without singularities. The latter is, however, inconvenient from the practical point of view, especially in the path integral formalism. The aim of the present paper is to develop an alternative approach where dynamics on the orbit space is described without gauge fixing for gauge fields.

2 Supersymmetric quantization.

Let us extend the initial configuration space $[\mathbf{A}]$ by adding to the system two complex scalar (ghost) fields: commutative ϕ and anticommutative (Grassmann) η ones [2]. We shall denote the set (ϕ, η) as Θ . The ghost fields realize a unitary representation of the gauge group

$$\Theta \rightarrow \Theta^U = T_U \Theta, \quad \Theta^* \rightarrow \Theta^{*U} = \Theta^* T_U^+, \quad (3)$$

where T_U is the group element U in the representation of the ghosts. Consider quantum dynamics governed by the extended Hamiltonian [3]

$$H = H_{YM} + H_{gh} = H_{YM} + [Q, R^+]_+ = H_{YM} + [Q^+, R]_+, \quad (4)$$

$$Q = i \int d^3x [(\eta^+, p_\phi) - (p_\eta^+, \phi)], \quad R = i \int d^3x [(p_\eta^+, p_\phi) + (\eta^+, M(\mathbf{A})\phi)]; \quad (5)$$

here $(,)$ stands for an invariant scalar product in the ghost representation space, p_Θ and p_Θ^+ are canonical momenta for Θ^+ and Θ , respectively, satisfying the same statistics as the ghosts (i.e., the bosonic ghost is quantized via a commutator, while the fermionic ghost via an anticommutator), and the linear, local and positive operator $M = M^+$ has the transformation property $M(\mathbf{A}^U) = T_U M(\mathbf{A}) T_U^+$, so that the extended Hamiltonian is gauge invariant. The operators (5) are nilpotent, $Q^2 = (Q^+)^2 = 0$, $R^2 = (R^+)^2 = 0$. The ghost dynamics exhibits an $N = 2$ supersymmetry. From (4), (5) follows that the ghost Hamiltonian H_{gh} , the odd operators Q , R and their adjoints form

the $N = 2$ superalgebra [3]. The operator M can be chosen, for instance, as $-\mathbf{D}^2$.

Let σ_{gh}^a be operators that generate the gauge transformations (3) of the ghosts. Gauge invariant states in the extended theory are determined by

$$\sigma^a \Psi_{ph}[\mathbf{A}, \Theta, \Theta^*] = (\sigma_{YM}^a + \sigma_{gh}^a) \Psi_{ph}[\mathbf{A}, \Theta, \Theta^*] = 0 . \quad (6)$$

Our main observation is that [3] the physical subspace of the initial gauge theory determined by (2) is isomorphic to a subspace of the physical Hilbert space in the extended theory that is selected by two supersymmetry conditions

$$Q \Psi_{ph}[\mathbf{A}, \Theta, \Theta^*] = Q^+ \Psi_{ph}[\mathbf{A}, \Theta, \Theta^*] = 0 , \quad (7)$$

modulo states of zero norm. Indeed, one can show that a generic solution to Eqs. (6) and (7) has the following form [3]

$$\Psi_{ph}[\mathbf{A}, \Theta, \Theta^*] = \Psi_{gh}^{vac}[\mathbf{A}, \Theta, \Theta^*] \Psi_D[\mathbf{A}] + \Psi_0 , \quad \Psi_0 = Q \Psi' = Q^+ \Psi'' , \quad (8)$$

where Ψ_{ph}^{vac} is the ghost vacuum determined by the equation $H_{gh} \Psi_{ph}^{vac} = 0$. Since H_{gh} is a Hamiltonian of an infinite dimensional supersymmetric oscillator, the latter equation always has a normalized unique solution. The norm of physical supersymmetric states is equal to the norm of the Dirac states in the initial gauge theory because i) Ψ_0 are orthogonal to the ghost vacuum (the ghost vacuum is supersymmetric) and ii) Ψ_0 have zero norm (the operators Q and Q^+ are nilpotent).

For any operator of the form $O = O_{YM} + [Q, O'] = O_{YM} + [Q^+, O'']$, where O_{YM} is independent of the ghosts, one can prove the relation [3] $\langle \Psi_{ph} | O | \Psi'_{ph} \rangle = \langle \Psi_D | O_{YM} | \Psi'_D \rangle$. In particular,

$$\langle \Psi_{ph} | e^{-itH} | \Psi'_{ph} \rangle = \langle \Psi_D | e^{-itH_{YM}} | \Psi'_D \rangle , \quad (9)$$

i.e., dynamics in the gauge invariant sector of the initial theory is equivalent to supersymmetric gauge-invariant dynamics of the extended theory.

3 Avoiding the Gribov problem.

Let $[\mathbf{A}, \Theta, \Theta^*]$ be the total configuration space of the extended theory. We have proved that the supersymmetric dynamics on the extended orbit space $[\mathbf{A}, \Theta, \Theta^*]_{ph} = [\mathbf{A}, \Theta, \Theta^*]/G$ is equivalent to dynamics on the gauge orbit space $[\mathbf{A}]_{ph}$. To resolve the aforementioned problem of parametrizing $[\mathbf{A}]_{ph}$, we introduce local coordinates on the extended orbit space and consider supersymmetric dynamics in these coordinates. The scalar bosonic ghost ϕ transforms homogeneously under the gauge transformations, therefore, the space $[\mathbf{A}, \Theta, \Theta^*]_{ph}$ can be parametrized by means of imposing an algebraic

gauge condition on it, $F(\phi) = 0$, that is, $[\mathbf{A}, \Theta, \Theta^*]_{ph} \sim [\mathbf{A}, \Theta, \Theta^*]_{F=0}$. The supersymmetry transformations generated by Q and Q^+ become non-linear and involve \mathbf{A} when projected on the hyperplane $[\mathbf{A}, \Theta, \Theta^*]_{F=0}$ [4].

Thus, the supersymmetric quantization scheme allows one i) to avoid the Gribov ambiguity in the gauge field sector (because there is no gauge condition imposed on gauge fields) and ii) leads to the Lorentz invariant path integral (a gauge condition is imposed on a Lorentz scalar ϕ) [2], [3].

On the canonical level the supersymmetric quantization has the further advantage that the scalar product is well defined and no regularization is required [3]. This is in contrast to the conventional BRST quantization based on gauge fixing[5].

The main obstacle in the implementation of the supersymmetric quantization is the difficulties presented by the renormalization of the theory. This problem has its origin in the fact that the supersymmetric quantization effectively corresponds to the choice of a unitary gauge[4].

4 Dynamical Abelian projection of gluodynamics.

Recent numerical simulations of the string tension in the lattice gluodynamics have shown that the main contribution to the Wilson loop average comes from some specific configurations of gauge fields [6]. These configurations appear to be monopoles after partial gauge fixing that restricts the gauge group to its maximal Abelian subgroup [7] and give about 93 percents of the total string tension [6]. This suggests that gluodynamics is equivalent to some effective monopole dynamics in the Abelian projection. To construct an effective Lorentz invariant action of monopoles in the continuum gluodynamics, one should, first, parametrize monopole configurations and, second, resolve the Gribov problem arising after the Abelian projection, which is difficult in the conventional approach. In the framework of the supersymmetric quantization, one can construct a new (dynamical) Abelian projection [8] where i) configurations relevant for the path integral (monopoles) are classified in a gauge invariant way and parametrized by a scalar (ghost) field in the adjoint representation, and ii) their effective dynamics can be derived (because the Gribov problem is no obstacle in this approach).

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